

**EXERCISES [MAI 4.9]**  
**DISCRETE DISTRIBUTIONS**  
**SOLUTIONS**  
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**A. Paper 1 questions (SHORT)**

1. (a) 2.35 (b) mode = 1 (c) median = 2 (d)  $Q_1 = 1$   $Q_3 = 3.5$   
 2.  $a = 0.2$   $b = 0.4$   
 3.  $E(X) = 2.2$   $E(\text{Profit}) = 0$ , so it is fair.  
 4. (a)

$x$	1	2	3
$P(X=x)$	1/6	2/6	3/6

- (b) Find  $E(X) = 7/3$   
 5. (a)

$x$	3	4	5
$P(X=x)$	4/10	3/10	3/10

- (b) Find  $E(X) = 3.9$   
 6. (a)  $(0.4 + p + 0.2 + 0.07 + 0.02 = 1)$ ,  $\Rightarrow p = 0.31$   
 (b)  $E(X) = 1(0.4) + 2(0.31) + 3(0.2) + 4(0.07) + 5(0.02) = 2$   
 7. (a)  $0.1 + a + 0.3 + b = 1 \Rightarrow a + b = 0.6$   
 (b)  $0 \times 0.1 + 1 \times a + 2 \times 0.3 + 3 \times b$   
 $0 + a + 0.6 + 3b = 1.5$   
 $a + 3b = 0.9$   
 Solving simultaneously gives  
 $a = 0.45$   $b = 0.15$   
 8. (a)  $0.2 + a + b + 0.25 = 1(a + b = 0.55)$   
 $E(X) = a + 2b + 0.75 = 1.55$   
 $\Rightarrow a + 2b = 0.8$   
 $a = 0.3$  and  $b = 0.25$

9.  $\sum_{\text{all } x} P(X=x) = 1 \Rightarrow \frac{1}{5} + \frac{2}{5} + \frac{1}{10} + x = 1 \Rightarrow x = \frac{3}{10}$

$P(\text{scoring six after two rolls}) = \left(\frac{1}{10} \times \frac{1}{10}\right) + 2 \times \left(\frac{2}{5} \times \frac{3}{10}\right) = \frac{1}{4}$

10. (a)  $10k^2 + 3k + 0.6 = 1 \Rightarrow 10k^2 + 3k - 0.4 = 0 \Leftrightarrow k = 0.1$  (by GDC)  
 (b)  $E(X) = -1 \times 0.2 + 2 \times 0.4 + 3 \times 0.3 = 1.5$

11. (a)

$x$	0	1	2	3	4
$P(X=x)$	$k$	$2k$	$3k$	$4k$	$5k$

- (b)  $k \times 1 + k \times 2 + k \times 3 + k \times 4 + k \times 5 = 15k = 1 \Leftrightarrow k = \frac{1}{15}$   
 (c)  $E(X) = 0 \times \frac{1}{15} + 1 \times \frac{2}{15} + 2 \times \frac{3}{15} + 3 \times \frac{4}{15} + 4 \times \frac{5}{15} = \frac{40}{15} = \frac{8}{3}$   
 12. (a)  $\sum P(X=x) = 1 \Rightarrow 4c + 6c + 6c + 4c = 1 \Rightarrow 20c = 1 \Rightarrow c = \frac{1}{20}$  (=0.05)  
 (b)  $E(X) = \sum xP(X=x) = (1 \times 0.2) + (2 \times 0.3) + (3 \times 0.3) + (4 \times 0.2) = 2.5$

$$13. \sum_{\text{all } x} P(X=x) = 1 \Rightarrow k + \frac{2}{3}k + \left(\frac{2}{3}\right)^2 k + \left(\frac{2}{3}\right)^3 k + \dots = 1$$

$$k \left( \frac{1}{1 - \frac{2}{3}} \right) = 1 \Rightarrow k = \frac{1}{3}$$

14. (a) Sum = 1.3 which is greater than 1

(b)  $3k + 0.7 = 1 \Rightarrow k = 0.1$

(c) (i)  $P(X=0) = \frac{0+1}{20} = \frac{1}{20}$

(ii)  $P(X>0) = 1 - P(X=0) = \frac{19}{20}$  (or  $\frac{4}{20} + \frac{5}{20} + \frac{10}{20}$ ) =  $\frac{19}{20}$

**Notice:** in fact, the probability distribution is

$x$	0	3	4	9
$P(X=x)$	1/20	4/20	5/20	10/20

15. (a) (i)  $1+2$  or  $2+1$  Prob =  $\frac{1}{2} \times \frac{1}{5} \times 2 = \frac{1}{5}$

(ii)  $1+3$  or  $3+1$  or  $2+2$  Prob =  $\frac{1}{2} \times \frac{1}{5} \times 2 + \frac{1}{5} \times \frac{1}{5} = \frac{1}{5} + \frac{1}{25} = \frac{6}{25}$

(b) Let  $X$  be the number of counters the player receives in return.

$$E(X) = \sum p(x) \times x = 9 \quad (\text{M1})$$

$$\Leftrightarrow \left(\frac{1}{2} \times 4\right) + \left(\frac{1}{5} \times 5\right) + \left(\frac{1}{5} \times 15\right) + \left(\frac{1}{10} \times n\right) = 9 \quad (\text{M1})(\text{A1})$$

$$\Leftrightarrow \frac{1}{10} n = 3 \Leftrightarrow n = 30 \quad (\text{A1})$$

### B. Paper 2 questions (LONG)

16. (a) Probability of two 4s is  $\frac{1}{16}$  (= 0.0625)

(b)

$x$	0	1	2
$P(X=x)$	$\frac{9}{16}$	$\frac{6}{16}$	$\frac{1}{16}$

(c)  $E(X) = \sum_0^2 xP(X=x) = 0 \times \frac{9}{16} + 1 \times \frac{6}{16} + 2 \times \frac{1}{16} = \frac{8}{16} = \frac{1}{2}$

or  $E(X) = np = 2 \times \frac{1}{4} = \frac{1}{2}$

(d)

$x$	0	1	2
amount	-2€	5€	20€
$P(X=x)$	$\frac{9}{16}$	$\frac{6}{16}$	$\frac{1}{16}$

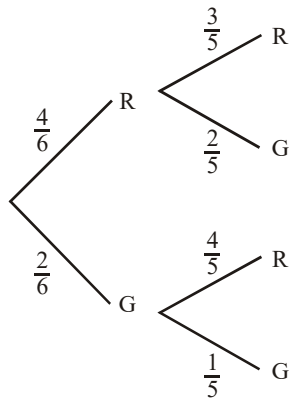
$$\text{Expected amount} = -2 \times \frac{9}{16} + 5 \times \frac{6}{16} + 20 \times \frac{1}{16} = 2€$$

(e)  $100 \times 2 = 200$  €

(f) 18 € implies 0 fours and 2 fours or vice versa

$$\frac{9}{16} \times \frac{1}{16} \times 2 = \frac{9}{128}$$

17. (a)  $E(X) = 0 \times \frac{3}{10} + 1 \times \frac{6}{10} + 2 \times \frac{1}{10} = \frac{8}{10}$  (0.8)  
 (b) (i)



- (ii)  $P(Y=0) = \frac{2}{5} \times \frac{1}{5} = \frac{2}{30}$   
 $P(Y=1) = P(RG) + P(GR) = \left( \frac{4}{6} \times \frac{2}{5} + \frac{2}{6} \times \frac{4}{5} \right) = \frac{16}{30}$   
 $P(Y=2) = \frac{4}{6} \times \frac{3}{5} = \frac{12}{30}$   
 Forming a distribution

$y$	0	1	2
$P(Y=y)$	$\frac{2}{30}$	$\frac{16}{30}$	$\frac{12}{30}$

- (c)  $P(RR) = \frac{1}{3} \times \frac{1}{10} + \frac{2}{3} \times \frac{12}{30} = \frac{27}{90} \left( \frac{3}{10}, 0.3 \right)$   
 (d)  $P(1 \text{ or } 6|RR) = P(A|RR) = \frac{P(A \cap RR)}{P(RR)} = \frac{1}{30} \div \frac{27}{90} = \frac{3}{27} \left( \frac{1}{9}, 0.111 \right)$

18. (a) (i)  $P(B) = \frac{3}{4}$  (ii)  $P(R) = \frac{1}{4}$

(b)  $p = \frac{3}{4}, s = \frac{1}{4}, t = \frac{3}{4}$

(c) (i)  $P(X=3) = P(\text{getting 1 and 2}) = \frac{1}{4} \times \frac{3}{4} = \frac{3}{16}$

(ii)  $P(X=2) = \frac{1}{4} \times \frac{1}{4} + \frac{3}{4} \left( \text{or } 1 - \frac{3}{16} \right) = \frac{13}{16}$

- (d) (i)

$X$	2	3
$P(X=x)$	$\frac{13}{16}$	$\frac{3}{16}$

(ii)  $E(X) = 2 \left( \frac{13}{16} \right) + 3 \left( \frac{3}{16} \right) = \frac{35}{16}$

- (e) win \$10  $\Rightarrow$  scores 3 one time, 2 other time

$P(\text{win } \$10) = P(3) \times P(2) + P(2) \times P(3) = 2 \left( \frac{13}{16} \times \frac{3}{16} \right) = \frac{78}{256} \left( = \frac{39}{128} \right)$

19. (a) 1, 2, 3, 4

(b)  $P(Y=1) = \frac{2}{5}$

$$P(Y=2) = \frac{3}{5} \times \frac{2}{4} = \frac{3}{10}$$

$$P(Y=3) = \frac{3}{5} \times \frac{2}{4} \times \frac{2}{3} = \frac{1}{5}$$

$$P(Y=4) = \frac{3}{5} \times \frac{2}{4} \times \frac{1}{3} \times \frac{2}{2} = \frac{1}{10}$$

(c)  $E(Y) = 1 \times \frac{2}{5} + 2 \times \frac{3}{10} + 3 \times \frac{1}{5} + 4 \times \frac{1}{10} = 2$

(d)

$X$	1	2	3
$P(X=x)$	3/5	3/10	1/10